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The Kinematic Car: Teaching Undergraduates Nonholonomic Mechanical System Basics

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Abstract

The project-based undergraduate J-term course *Kinematics and Mechanism Design* is described, both technically and from a pedagogy point of view. In this course students discovered the kinematic car, a classic example of a nonholonomic mechanical system that everyone can relate to; especially mechanical engineering students. Technical work entailed kinematic modeling using MATLAB®/SIMULINK® and CAD modeling and visualization using SolidWorks® along with corroborating experimental work using scale model vehicles (i.e. Jeep Liberty SUV, Allis-Chalmers WC tractor). Pedagogy issues pertain largely to the use of integrated project-based learning that incorporated a variety of technical tasks (theory, CAD modeling, kinematic simulation, and experimental work), teamwork, inquiry-based learning, and documentation, including presentation to the class.

Keywords: nonholonomic mechanical systems, project and inquiry-based learning, CAD modeling, kinematics

1. Introduction

January (or "J") - term presents unique opportunities for offering non-traditional, or specialty courses, in part because of its short duration (4 weeks typical) and usually small class sizes. Within the Engineering Department at the University of St. Thomas, for each of the past 3 years, *Kinematics and Mechanism Design* (ENGR 225) [1] has been offered and has focused on a different kinematics project each year; typically associated with the author's current research interests. Three years ago the project was in robot kinematics [2], 2 years ago slop in rigid-part mechanical assemblies was studied [3,4], and this past year the focus was on kinematic analysis of a classic nonholonomic mechanical system, i.e. the kinematic car. In all of the above cases, project and inquiry-based learning played a critical role in the success of the course. Finally, historically the course has been designed to support participation by students in an undergraduate conference, once again this was the case this past year [5].

Course Description: Analysis and design of linkages and other mechanisms including geometry of motion and force distributions. Computer aided analysis and design tools are used as well as mathematical techniques.

Objective: This J-term course is research project-based and will focus on: (1) acquiring expertise in computer graphics using SolidWorks® [6-8], (2) acquiring expertise in modeling the kinematics and control of mechanical systems using MATLAB®/SIMULINK® [9,10], and (3) working on an applied research project in the area of nonholonomic mechanical systems.

Project Overview: A team of approximately 5 students will work together on a project involving the open-loop motion of a nonholonomic mechanical system, specifically a simulated vehicle moving slowly in a simulated parking lot, where "no-slip" wheel conditions apply. Several scale model vehicles will be made available for this express purpose (note: if you can find another more interesting vehicle you are welcome to do so, subject to instructor approval). The project will rely heavily on computer software (MATLAB®, SIMULINK®, SolidWorks®, Power Point, and possibly Adobe Illustrator), experimental work, documentation generation, and teamwork.

2. Kinematic Car Model

A classic example of a nonholonomic[1] mechanical system is that of a wheeled vehicle, i.e. the so-called "kinematic car" illustrated in Figure 1. Differential equations of motion written in control input form are given by [11]:

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} \cos\theta \\ \sin\theta \\ \dfrac{\tan\phi}{L} \\ 0 \end{bmatrix} u_2 \tag{1}$$

or $\dot{\boldsymbol{q}} = \boldsymbol{g}_1 u_1 + \boldsymbol{g}_2 u_2$, where $\boldsymbol{q} = [x\ y\ \theta\ \phi]'$ represents the system state; the inputs to the system are the steered wheel angle rate ($u_1 = \dot{\phi}$) and the forward velocity of the rear axle center (u_2), and $\boldsymbol{g}_1, \boldsymbol{g}_2$ are implied from context. Equation (1) can be integrated numerically using a simulation diagram with 4 integrators and SIMULINK® as depicted in Figure 2 for the case of a specific choice of inputs. Different kinematic maneuvers were studied: (1) circular maneuver, (2) "Lie bracket" maneuver, and (3) parallel parking maneuver.

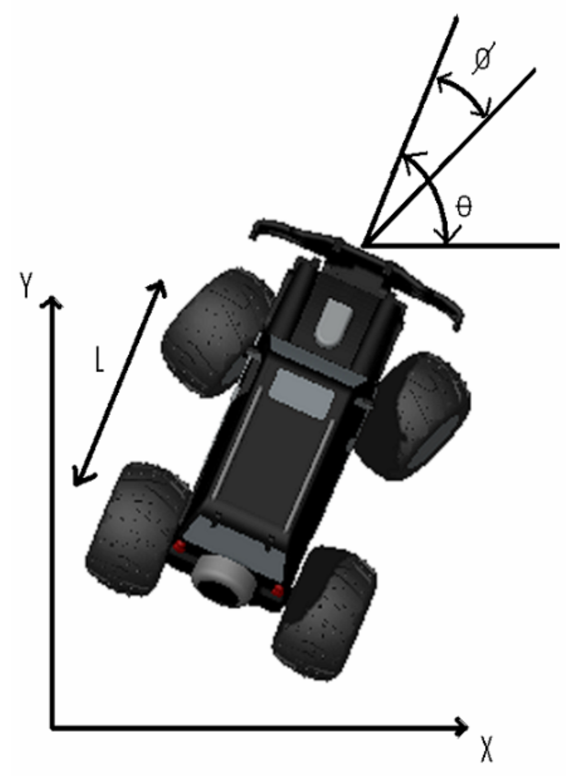

Figure 1. *Planar kinematic car model with x, y, θ, ϕ state variables; (x, y) – position of the center of the rear axle, θ – vehicle heading angle, and ϕ – steered wheel angle (- as shown, with range of +/- 90 degrees).*

Circular Maneuver: The circular maneuver is the simplest of those studied. For a circular maneuver it can be shown that the radius (ρ) of the resulting circular trajectory with $\phi = C$ (constant) obeys the following equation: $\rho = L \big/ \tan|\phi|$.

Lie-Bracket Maneuver: The so-called "Lie-bracket [11]" maneuver is a 4-step maneuver (each step is of duration ε sec), cycling through the 2 inputs (steered wheel angle rate (+/-), forward velocity of the rear axle center (+/-)) and returning to the original input state, yet the system can be offset from where it was initially, before the maneuver. To be more precise, for truly incremental motion, to second order in ε the resulting state offset is given by:

$$\varepsilon^2 \bar{u}_1 \bar{u}_2 [\boldsymbol{g}_1, \boldsymbol{g}_2] = \varepsilon^2 \bar{u}_1 \bar{u}_2 [0 \;\; 0 \;\; \frac{\sec^2 \phi}{L} \;\; 0]' \quad (2)$$

where $\bar{u}_1, \bar{u}_2$ are constant inputs and $[\boldsymbol{g}_1, \boldsymbol{g}_2]$ is the Lie bracket of the 2 vector fields $\boldsymbol{g}_1, \boldsymbol{g}_2$ given by:

$$[\boldsymbol{g}_1, \boldsymbol{g}_2] = \frac{\partial \boldsymbol{g}_2}{\partial \boldsymbol{q}} \boldsymbol{g}_1 - \frac{\partial \boldsymbol{g}_1}{\partial \boldsymbol{q}} \boldsymbol{g}_2 \quad (3)$$

A nonzero Lie-bracket is a manifestation of the non-commutivity of the 2 input motions. The above calculation indicates that a heading angle (i.e. θ) change is induced. Of course for non-incremental motion (i.e. non-incremental ε), the position (x, y) will change as well.

Parallel Parking Maneuver: The parallel parking maneuver was by far the most interesting from a pedagogy point of view. Rigorously solving this problem requires graduate coursework in applied mathematics/engineering analysis pertaining to nonholonomic mechanical systems; yet as we all know, people lacking such knowledge routinely parallel park their cars reliably on a daily basis. Similarly, students were in the position of "let's try this and see what happens," etc. as they achieved a practical solution to the problem.

3. Student Activities

Given the kinematic car model theory, student teams participated in the following activities: (1) computer-aided-design (CAD) modeling, (2) kinematic modeling using MATLAB®/SIMULINK®, and (3) experimental validation, in addition to proper documentation including a poster and a Power Point presentation given to the class. Two teams of 6 and 4 students each respectively, were formed, one used a scale model radio-controlled (RC) vehicle (a Jeep Liberty sport utility vehicle (SUV) -- see Figure 3) and the other used a Franklin Mint scale model Allis-Chalmers WC tractor (see Figure 4).

Most if not all of the students had previously taken *Engineering Graphics* (ENGR 171) based largely on SolidWorks® and welcomed the challenge that the model vehicle geometry presented [12]. Many CAD techniques from the course were used to approximate the geometry of the vehicles provided (Jeep Liberty SUV and Allis-Chalmers WC tractor). Some of the more advanced CAD techniques included sweeping and lofting (especially useful for highly sculpted shapes), coloring/transparency techniques, plus proper assembly/subassembly/part/feature structure to support assembly configurations that easily convey useful motions.

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[1]To summarize, if a constraint $A(\boldsymbol{q})\dot{\boldsymbol{q}} = 0$ with $\boldsymbol{q} \in Q$ can be written in the form $(\partial h / \partial \boldsymbol{q})\dot{\boldsymbol{q}} = 0$ where $h : Q \to R^k$, the constraint is said to be holonomic; if not, the constraint is nonholonomic.

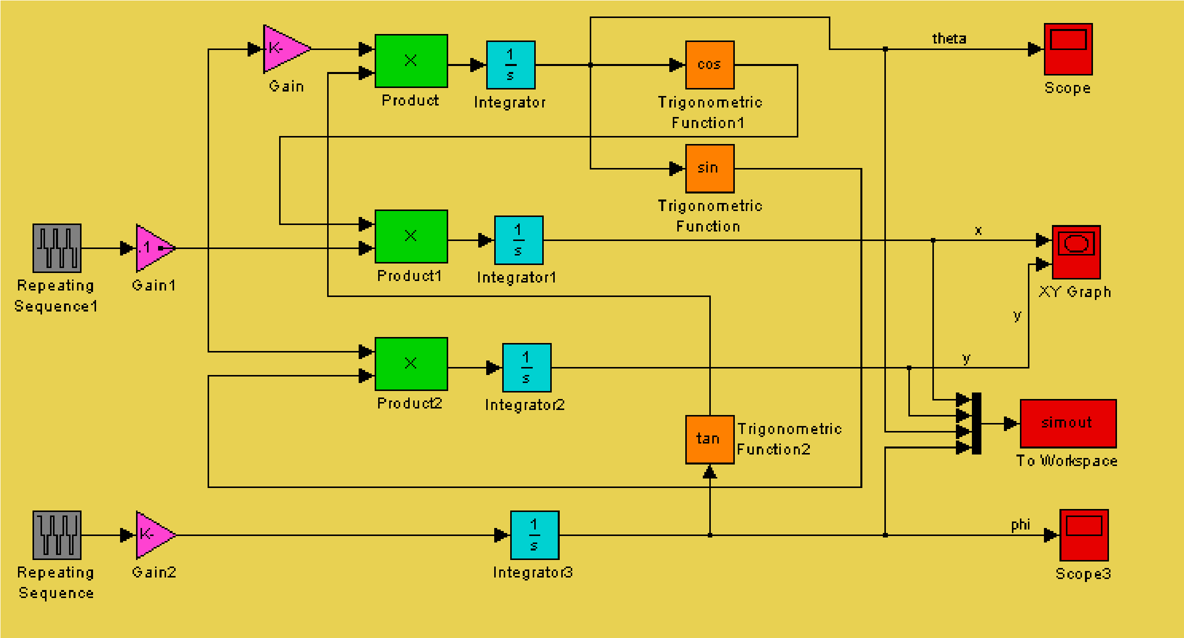

Figure 2. *SIMULINK® model (20 blocks including 4 integrators) of kinematic car.*

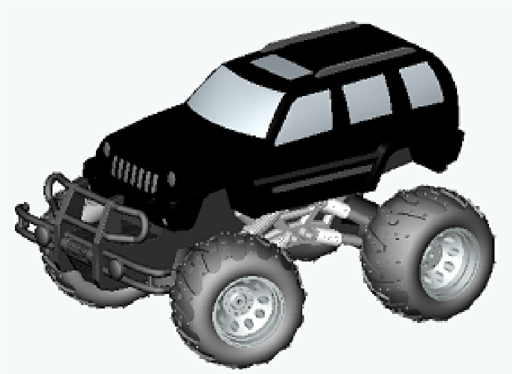

Figure 3. *CAD model (SolidWorks®) of RC model Jeep Liberty SUV.*

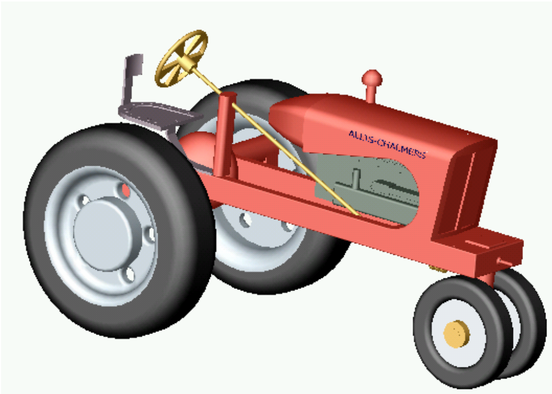

Figure 4. *CAD model (SolidWorks®) of scale model Allis-Chalmers WC tractor.*

Kinematic modeling using MATLAB®/SIMULINK® was based upon the analytical model provided (i.e. Eqn. (1)) and applied it to both the Jeep Liberty SUV and the Allis-Chalmers WC tractor. Assuming Ackerman steering [13], the only critical dimension is L (the wheel base dimension) and simulations were performed for each of the 3 suggested kinematic maneuvers mentioned above. The major challenge was to get good correspondence between the theory/simulation and experimental data. For the students, typically this required several iterations, where mistakes were found and corrected, and greater attention was paid to detail.

Experimental validation was critical to convincing students that the theory/simulation is actually correct. All three maneuvers mentioned above were validated experimentally. Figures 5 and 6 illustrate the "circular maneuver" simulation and experimental validation for the case of the Allis-Chalmers WC tractor. Note that both theoretical/simulation and experimental data were fed into the SolidWorks® environment (manually). Figures 7 and 8 illustrate a Jeep Liberty SUV performing the "Lie bracket" maneuver both through theory/simulation and experimentally. It is worth mentioning that the student generated simulations and display techniques helped identify useful plotting techniques applicable to the author's current research [14]. To properly interpret Figure 7, observe that while both theoretical/simulation and experimental data sets are depicted, they are offset by a 180 degree rotation (about z axis) to minimize overlaps and enhance clarity/understanding. Note also the use of what we term "variable ghosting (or transparency)," in SolidWorks® to attempt to illustrate the effect of motion. Figure 9 illustrates an effective parallel parking maneuver performed by the Jeep Liberty SUV. Through simulation and experimentation, students discovered a practical 2-step maneuver where during each step the steered wheel angle was fixed.

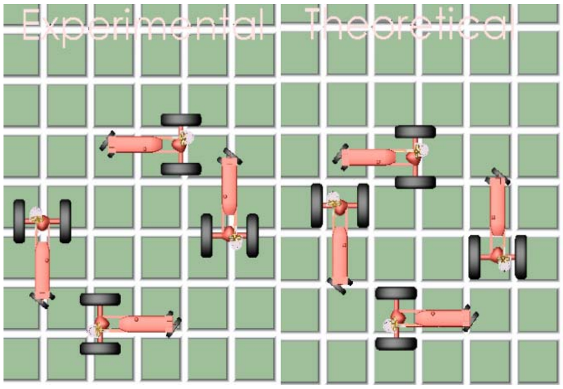

Figure 5. *Illustration (SolidWorks®) of circular maneuver performed by scale model Allis-Chalmers WC tractor; both theoretical and experimental data.*

Figure 6. *Scale model Allis-Chalmers WC tractor performing a circular maneuver.*

4. Inquiry-Based Learning and Benefit to Course

General course objectives were given to the students (i.e. course description, objective, and project overview from above plus a few short lectures with handouts) and they had an opportunity to fill in the details on their own making various decisions and trying out different ideas. This was especially true for the CAD work, experimental work, and figuring out how to parallel park their vehicles. CAD modeling provides a wonderful opportunity for inquiry-based learning. Students make decisions regarding how to prioritize and best approximate the geometry of various features/parts in the assembly. Different approaches are tried within the CAD package (i.e. SolidWorks®) and in the end a reasonable fidelity model is obtained. Experimental work conducted entailed modifying the vehicles (using the machine shop as necessary) to minimize backlash in the front wheels (thereby improving accuracy), devising a means of measuring the various states (i.e. x, y, θ, ϕ), setting up an experimental environment including a simulated parking lot, and lastly, performing experiments that corroborated the theory/simulation for the different scenarios/maneuvers suggested. The parallel parking maneuver activity was ideally suited to inquiry-based learning. The goal was clear, however, no specific instructions were ever given. Students tried different ideas both in the simulation environment (i.e. MATLAB®/SIMULINK®) as well as using the scale model vehicles until a reasonable solution was discovered based on a time sequence of constant steered wheel angle inputs. Effective teamwork was also essential to achieving success.

5. Conclusions

An integrated project-based learning approach can be very effective when applied to non-traditional courses such as those taught during J-term. Such was the case for *Kinematics and Mechanism Design* that incorporated the following:

- Variety of essential technical tasks (theory, CAD modeling, simulation, and experimental work)
- Teamwork
- Inquiry-based learning
- Documentation, including presentation to the class

The kinematic car project was ideally suited to integrated project-based learning, with each of the above elements incorporated in a non-trivial way.

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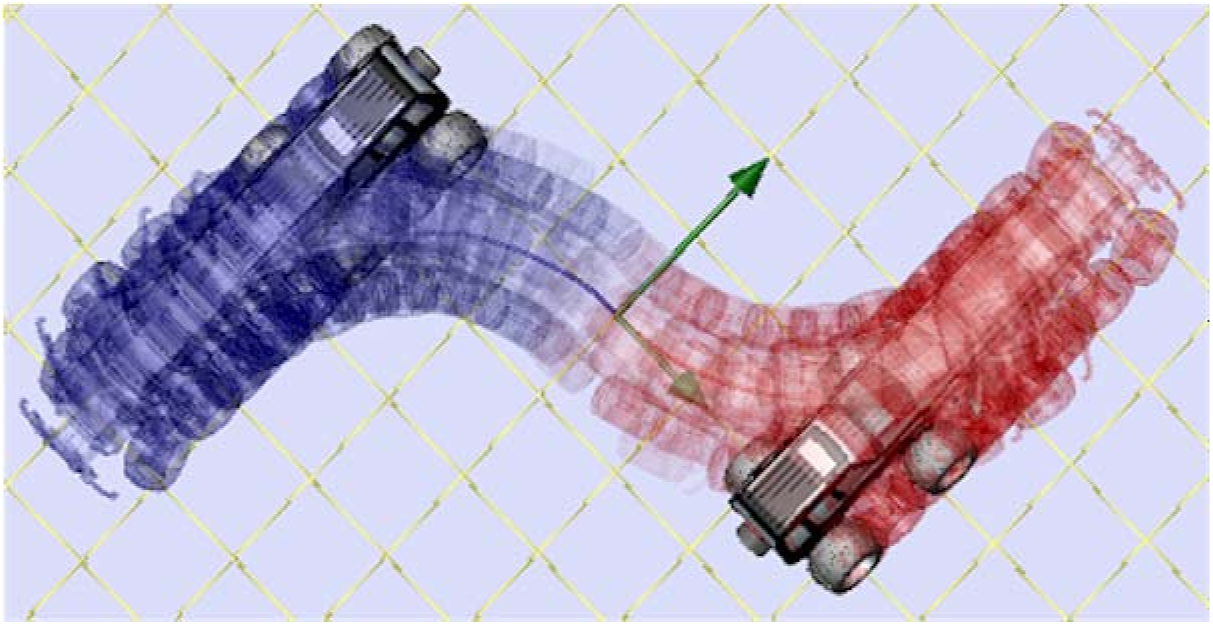

Figure 7. *Artistic illustration (SolidWorks®) of non-incremental Lie bracket maneuver performed by RC model Jeep Liberty SUV; both theoretical (blue, mostly on left) and experimental (red, mostly on right) data sets are shown with 180 degree offset.*

Figure 8. *RC model Jeep Liberty SUV performing a non-incremental Lie bracket maneuver.*

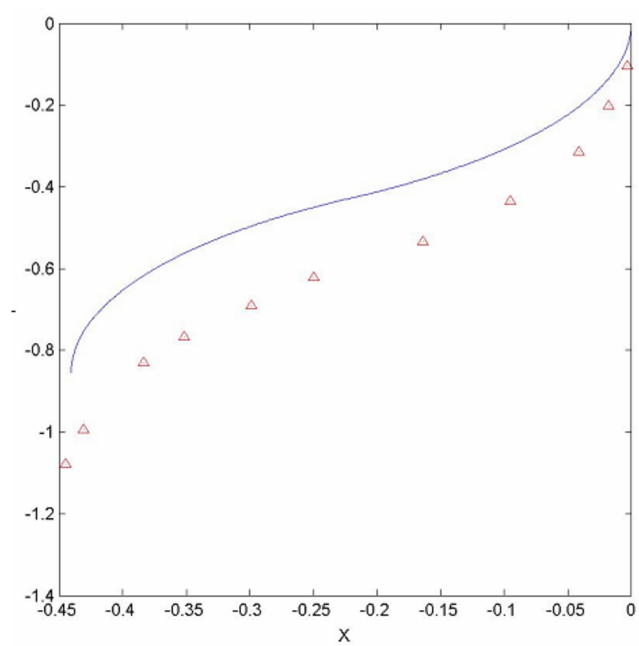

Figure 9. *Parallel parking maneuver performed by RC model Jeep Liberty SUV – (x, y) data (theoretical – blue curve, experimental – purple triangles).*

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