Integrating Modeling and Numerical Analysis into Systems-Level Design

Joseph C. Musto and Vincent C. Prantil
Mechanical Engineering Department
Milwaukee School of Engineering

Abstract

The development of systems-level design experiences is a challenge in the design of an undergraduate Mechanical Engineering curriculum. One key challenge is the coupling of the more analytical coursework component of the curriculum with creative design experiences. Traditional capstone design experiences involve a great deal of creative design, prototyping, debugging, and testing, but are sometimes weak in the use of mathematical modeling and computer simulation. This paper details a unique course experience developed in the Mechanical Engineering Program at Milwaukee School of Engineering that integrates topics from a traditional modeling/numerical methods course into a systems-level design project. The term-long design effort incorporates a structure where student design teams are led through a complex systems-level modeling exercise, and then use their mathematical model to optimize the design of a complex system. Both the philosophy of course development and example project applications are presented. Conclusions are presented indicating that both an increased understanding of theoretical aspects of modeling and an increased appreciation for the role of modeling and simulation in engineering design are accomplished through the course structure.

I. Introduction

The integration of design experiences is a major challenge in the development of a Mechanical Engineering curriculum. Design experiences provide an opportunity for students to extend their analytic experiences to the creative synthesis of solutions to open-ended problems. The design experiences in many traditional Mechanical Engineering programs can be generalized as follows:

- *Component-Level Design Experiences:* These experiences are generally encountered as course-integrated design experiences. Examples of such design experiences may include:
    - A design project in a heat transfer course, where students "design" a heat exchanger by selecting important dimensional quantities to optimize the cooling of a circuit board.
    - A design project in a machine components course, where students select appropriate spur gears to a specified deliver shaft power.
    - A materials selection problem in a materials science course, where students select an alloy to optimize a casting process.

These are valuable components of a Mechanical Engineering education; they offer the opportunity to apply analytic techniques to open-ended problems, and introduce the students to concepts of design optimization. However, they are often limited to parametric design problems; they involve only dimensional synthesis, and provide only limited opportunity to consider radically different design alternatives and exercise engineering creativity. In addition, design problem descriptions are often limited to very specific performance specifications, and larger "systems-level" issues are generally not addressed.

- *Systems-Level Design Experiences:* These experiences are generally not integrated into courses, but are offered as "stand-alone" design projects. While most "Capstone" design experiences would fall under this classification, such experiences may also be used as introductory experiences for freshmen students. In such experiences, students employ a formal design process, and generally take a project from a "recognition of need" phase to some form of design realization. In many of these experiences, there is a strong focus of problem definition, development of design specifications, generation of design alternatives, consideration of realistic constraints, and design realization. These projects may range from a single academic term to an entire academic year. Such projects can provide a realistic academic simulation of an industrial project, and allow for significant student creativity; however, when allowed to work in the unstructured environment that characterizes such design efforts, students often focus more on the prototyping, testing, and intuition that characterize a "build-it-and-break-it" approach to design, rather than employing the analytic skills acquired in academic coursework.

At Milwaukee School of Engineering (MSOE), a unique design experience is used to attempt to merge the more analytically-focused parametric "component" design experiences with the highly-creative systems-level design experiences. This design experience is part of a two-course sequence in numerical methods; by integrating a systems-level design effort into a traditional numerical methods sequence, techniques for mathematical modeling and simulation can be more readily infused into the design effort, allowing for a highly analytic approach to systems-level design and optimization.

The course sequence used to accomplish this is a two-course Junior level sequence. The first course is a four-credit, quarter-long course entitled *Modeling and Numerical Analysis*. This is a traditional numerical methods course, including topics such as matrix computation, root finding, numerical integration, and numerical solution of differential equations [1]. This course also includes a laboratory component, where modeling and numerical solution techniques are applied to a variety of mechanical and thermal systems (such as a draining bottle, a heated and quenched object, and a spring-mass-damper system). Student feedback from this numerical methods course indicates two negative aspects to this traditional approach to teaching numerical methods:

- Despite the presence of a laboratory component, students cite a lack of connection between the highly mathematical course topics and practical physical applications.
- Students fail to see the interconnection between the seemingly diverse concepts seen each week.

The second course in the sequence is a three-credit, quarter-long course entitled *Computer Aided Engineering*. In this course, topics from the *Modeling and Numerical Analysis* are applied to a complex systems level design project. Over the term, a mathematical model of a complex system is formulated; the project is structured such that a variety of modeling and numerical solution techniques are employed in the development of the system model. Once the mathematical model is constructed, programmed, debugged, and tested, the model is used by student design teams for design optimization. The use of these techniques to construct a system model and optimize the design of a physical system overcomes the two weaknesses cited in the traditional numerical methods course [2]; in addition, it provides a strong experience in the integration of modeling and numerical analysis into a system-level mechanical engineering design project.

In Section II of this paper, a detailed description of the course structure will be provided. In addition, two sample applications developed during the 2001-2002 academic year will be provided. In Section III, conclusions and recommendations for future implementations of this type of project-based course will be offered.

II. Course/Project Structure

The goal of the project-based numerical methods course is to allow the students to apply modeling and simulation techniques to the design of a complex system. *Matlab* was selected as the programming tool; this was done based on both student familiarity and availability. The course is structured such that the first two-thirds of the term are devoted to the construction, programming, debugging, and testing of a mathematical model of a complex system, leaving the remaining portion of the course for design optimization and documentation.

The task of construction a high-fidelity simulation model and system design of a complex mechanical or thermal system is a new challenge for most students at the Junior level; for most, their modeling, simulation, and design experiences have been limited to individual components. The approach to system modeling that is taken in this course is to decompose the system model into subcomponent models; the subcomponent modeling tasks take the form of weekly project assignments. Students are responsible for modeling and simulating one subcomponent of their system each week, as well as for integrating each weekly submodule with those developed in previous weeks. After six or seven such weekly assignments, students have developed a complex system model from their subcomponent models; after completing all assignments, students have both a firm grasp of the performance of their subcomponents, as well as an understanding of how those subcomponents interact to determine the behavior of a complex system.

The design component of the project comprises the remaining three to four weeks of the term. Students are presented with a project description; they are then required to perform the following tasks:

- Develop a set of performance objectives for their system.
- Develop an objective function, which accurately characterizes the qualities of the system they are attempting to optimize.
- Perform a parametric design optimization, using the simulation model developed in the preceding part of the term.
- Present their results, both in a formal design report and in an oral presentation.

Developing a course project of this nature is a significant challenge for an instructor. An ideal project of this nature should include the following attributes:

- The system must be composed of no more than five or six interconnected subcomponents, to allow for the project to be completed in a 10-week term.
- The subcomponent modeling and simulation should require the students to exercise a wide variety of numerical solution techniques.
- The physical concepts required to develop the subcomponent models should be within the grasp of undergraduate students.
- The system model should contain some nonlinearities, to demonstrate the complexity of simulation and optimization in nonlinear systems.
- The development of the system should require interdisciplinary knowledge

(thermal/mechanical, electrical/mechanical, biological/mechanical, etc.).

In the remainder of this section, two example applications used during the 2001-2002 academic year at MSOE will be detailed. One involves the design of a robotic assembly system, involving significant electro-mechanical modeling and simulation. The other involves the design of a depth profile for repetitive SCUBA dives, and involves both thermo-fluidic and biological modeling and simulation.

II.A: Design of a Robotic Workcell [2]: In this project, students begin by developing a simulation model of a computer-controlled SCARA manipulator. The subcomponent modeling consists of five assignments, decomposed as follows:

- *Assignment 1 -- Development and Solution of Forward and Inverse Kinematic Models:* This requires the use of physical modeling techniques from machine dynamics, as well as implementation of the Newton-Raphson method for solving nonlinear simultaneous equations for the system shown in Figure 1:

$$x = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2)$$
$$y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2)$$

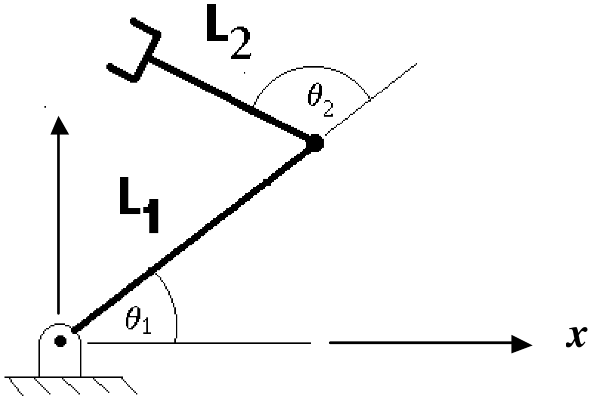

Figure 1: Schematic of the Robot

- *Assignment 2 -- Development of a Trajectory Plan:* This phase of the project involves the planning of a robot arm movement from one desired point in space to another. Manipulator joint angles for the desired endpoints are determined using the output from the simulation model developed in Assignment 1; the manipulator trajectory is then determined using a numerical technique for polynomial spline curve fitting:

$$\theta_i(t) = a_o + a_1 t + a_2 t^2 + a_3 t^3$$

- *Assignment 3 -- Generation of Manipulator Dynamics:* Based on the manipulator trajectories developed in Assignment 2, manipulator dynamics are determined by numerically differentiating the position trajectories:

$$\square_i = \frac{d\square_i}{dt}$$
$$\square_i = \frac{d\square_i}{dt}$$

In addition, simplified torque equations are used to determine the motor torques required to produce the desired motion [3].

- *Assignment 4 -- Open-Loop Manipulator Simulation:* In this assignment, the torque profiles developed in Assignment Three are used to develop a simulation of the performance of the manipulator under open-loop control. Both random and deterministic errors are introduced into the model to simulate the effect that mass misestimates, inaccurate position estimates, and other kinematic/geometric uncertainties have on open-loop performance. Euler's Method is employed in the numerical integration of the system dynamic equations to perform the manipulator simulation:

$$\square_1(t + \square t) = \square_1(t) + \square_1(t)\square t$$
$$\square_2(t + \square t) = \square_2(t) + \square_2(t)\square t$$
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$$\square_2(t + \square t) = \square_2(t) + \square_2(t)\square t$$

- *Assignment 5 -- Closed-Loop Manipulator Simulation:* In this phase of the project, a simulation of a closed-loop computed torque controller is added to the simulation model. The closed-loop controller is shown to mitigate the effect of kinematic/geometric uncertainties in the performance of the manipulator. Again, Euler's Method is used to integrate the equations of motion and perform the robotic simulation.

At the conclusion of there five assignments, the students have a working simulation model of a SCARA robot arm operating under closed-loop control. They are then presented with a design task.

- *Design Assignment:* Design a robotic workcell for pump assembly consisting of a pair of robot arms and a pair of conveyor belts. The goal is to design the robotic arms and configure the workcell to minimize the cycle time for pump assembly. Design variables for this systems-level design project include:

  - The length of the robot links.
  - The cross-section size/shape of the robot links.
  - The materials used in the robot.
  - The motors/transmission components used in the robot arms.
  - The location of the robots and conveyors in the workcell.

  Objective functions for design optimization may include the following design/performance objectives:

  - Total system cycle time.
  - System energy consumption.
  - System cost.
  - Overall workcell size.

System modeling was performed by individual students, but the design task was performed in teams of two or three students. The deliverables of the project included:

- A detailed list of design specifications
- A summary of the students' modeling, ***including any assumptions made***
- An overview of student team design calculations
- A description of the various *design iterations* evaluated
- A detailed description of the team design, including ***dimensioned design drawings*** of the robot, a workspace layout drawing, manufacturer's specifications for motors, etc.
- All relevant plots (joint trajectories, robot hand motions, motor power consumption, etc.)
- Any recommended design changes to the pump components or workcell.

II.B: Design of a Dive Profile for Multiple Dives: In this projects, students begin by developing simulation models for a dive computer using the thermo-fluidic and biological processes that govern the onset of decompression sickness (DCS) or the so-called *bends* during SCUBA diving [4].

- *Assignment 1 -- Modeling and Simulation of Tissue Half-Times:* The partial pressure of inert nitrogen gas in the bodily tissues of SCUBA divers submerged beneath water exhibits asymptotic saturation characteristic of a classical first-order system. For this system, the differential equation governing the tissue pressure, $p(t)$, is

$$\frac{dp(t)}{dt} + \square p(t) = \square p_{AMB}(t)$$

where $\square$ is a material parameter that determines the time constant for the system response and $p_{AMB}(t)$ is the ambient pressure in the water surrounding the diver. For a constant ambient pressure, the solution attributed to Haldane [5] is well known

$$p(t) = p_{AMB} + (p_O \square p_{AMB})e^{\square\square t}$$

Students are given data for nitrogen pressure histories in two body tissues with different time constants (as shown in Figure 2), and asked to determine the constant $\square$ through the use of numerical curve fitting algorithms.

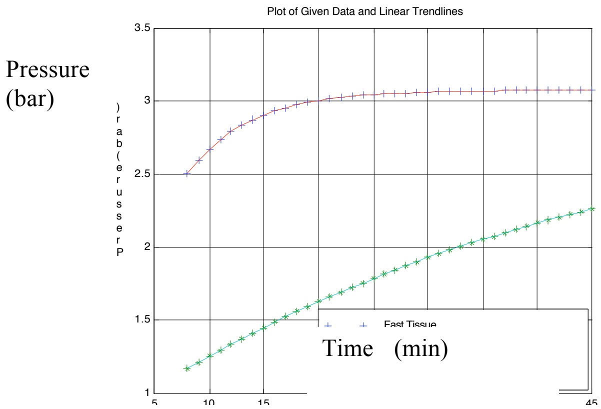

Figure 2: Linear Regression Curves

- *Assignment 2 -- Determination of No-Decompression Limits:* In this assignment, students utilize the Workman Decompression Model [6] to model the response of human tissue to the pressure changes associate with an underwater dive. A Newton-Raphson root finding algorithm is employed to determine the time at which the pressure would exceed the maximum value limits (called M values) associated with the six different tissue compartments that characterize the human body. Immediate ascent of tissues beyond their maximum limit must be avoided in order that the diver not suffer symptoms of DCS.

- *Assignment 3 -- Modeling of Repetitive Diving Profiles:* Square well dive profiles are typically logged by divers as illustrated in Figure 3. Dive tables and, more recently, dive computers are used to simulate nitrogen uptake on dives and plan time limits for the diver at a series of depths. Students are tasked with extending their module to retain the initial tissue pressures from the previous dive, predict the out-gassing at the surface between dives, and calculate the continual gas uptake on the second dive. The result is always a reduced no-decompression time at depth for the second dive. This is due to the presence of residual nitrogen from the previous dives. Certified divers are taught to plan reduced dive times based on the amount of residual nitrogen time (RNT) caused by tissue pressure remaining from previous dives done the same day. When this module is complete, students have coded the first elements of a rudimentary dive computer.

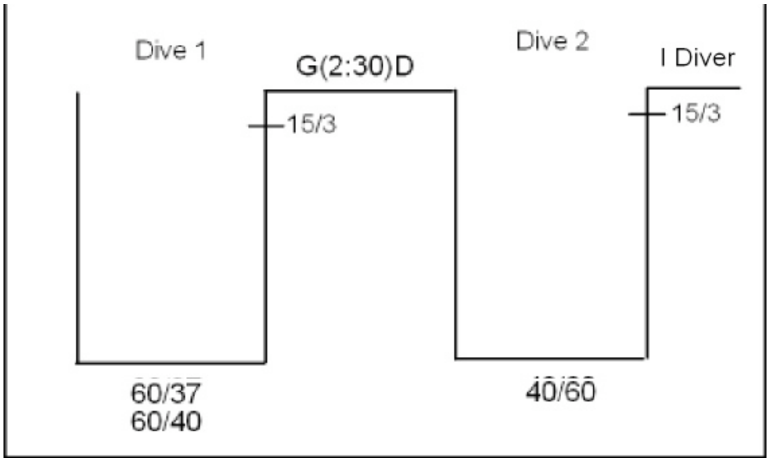

Figure 3: Square Well Dive Profiles

As part of the course, a special, small-scale pressurized chamber was built in MSOE's Fluid Power Institute. An actual nitrogen diffusion-based dive computer (1996 Oceanic Prodigy Model) was tested to perform a series of dives in succession, each to a different depth. Various pressure histories can be manually controlled. Three specific dive profiles were simulated in the lab and assigned to the students. Although the algorithm for the actual dive computer is not published, students are able to see that their algorithms produce no-decompression time limits that are reasonably close to commercially calculated dive scenarios.

- *Assignment 4 -- Modeling of Constant Speed Descents and Ascents*: To this point, we have examined only idealized dive profiles. Namely, dive depth is constant and divers submerge and ascend instantaneously. Divers can plan such profiles by using look-up dive tables that chart the results of tissue pressure at discrete depths and times. The goal of a dive computer is to continuously update the tissue pressure under an arbitrarily changing depth profile. As a first step in this direction and because they represent a very good approximation to actual diving conditions, the solution for pressure during constant speed descent and ascent is derived. Under such conditions the ambient water pressure is a linear function of time. An analytical expression for the tissue pressure diving in ocean salt water has been attributed to Schreiner [7] in which

$$P(t) = \left[P_0 - P_{Ao} + \left(\frac{0.03048K}{\lambda}\right)\right]e^{-\lambda t} + 0.03048Kt + P_{Ao} - \left(\frac{0.03048K}{\lambda}\right)$$

where P_{Ao} is the initial ambient pressure at the start of the depth change and K is the speed of descent. This solution will provide students a result with which to compare their numerically integrated pressures in the following weeks. With depth changes now continuous, the ambient pressure history is also continuous, resulting in no step changes in the slope of the individual tissue pressures.

Students are given several single and repetitive dive profiles with divers traveling at constant speed and given the task of, once again, determining no-decompression time limits and maximum tissue pressures. While a somewhat more complex exact solution is available for tissue pressure, there is no longer an analytic expression for the no-decompression time limit. In addition, students confronted with the interesting result that when the diver does not surface instantaneously, the relatively slow tissues might still be on-gassing during a significant portion of their ascent.

- *Assignment 5 -- Modeling of Instantaneous and Continuous Descent Rates:* At this point, students have simulated single and repetitive dive profiles under ideal conditions. Details of how to determine the onset of decompression sickness are known. Real dive computers, however, must be capable of continuously integrating the compartment tissue pressures in as many as 16 to 32 tissue compartments simultaneously. Real dive computers must also operate for arbitrarily varying ambient pressures, i.e. for an arbitrary depth profile. The next step is to introduce this generality into the scenario. For the purposes of our project, the ambient water pressure was introduced as a time-dependent boundary condition to the algorithm. This is accomplished in actual dive computers through a pressure transducer built into the hardware. Currently measured ambient pressures are used to calculate depth histories that are continuously displayed on the screen readout.

  Students are required to simulate tissue pressure histories from assignments 3 and 4 by continuously integrating the governing differential equation using a 4th order Runge-Kutta algorithm. Now the forcing function is an explicit, but otherwise arbitrary function of time:

  $$\frac{dp(t)}{dt} + \square p(t) = \square p_{AMB}(t) = \square F(t)$$

  Because the primary purpose of a dive computer is to specify the no-decompression time limit to the nearest minute, students are faced with situations in which the time step necessary for accurate tissue pressure resolution can be an order of magnitude larger than that required to resolve the time limit. In this way, students are required to look beyond the accuracy of the numerical method to practical application of the device when choosing bounds on their algorithmic step size.

- *Assignment 6 -- Modeling of Staged Decompression:* Students are introduced to inherent uncertainties in attempts to predict DCS. These are built into the fact that maximum limit pressures for the tissue compartments, although developed through hyper-baric trials with relatively large numbers of participants, cannot represent every individual's propensity to suffer symptoms of DCS. As such, relative uncertainties in the algorithmic M-values are built-in by artificially decreasing their absolute values while accounting for the observation that their effective values vary with depth. To model dive profiles realistically, students were required to plot the continuous tissue pressure histories for a series of cases in which the maximum tissue pressure limits were decreased systematically by as much as 10%, building in a factor of safety whereby the predictions will necessarily be conservative. The students were required to test their dive computer algorithm for the identical dive profile from Assignment #4, a single dive well with constant speed descent and ascent. The simulation is performed using a numerical integration algorithm.

Students were also tasked with generalizing the decompression model by systematically reducing the critical pressure values for each tissue with increasing depth once the diver has surpassed their decompression limit. In this scenario, tissue pressures can exceed the no-decompression limit when the diver has remained submerged too long to be able to return to the surface immediately. The diver must then be warned that they must decompress at some shallower depth below the surface before exiting the water. Students must generalize their module to determine a series of successive shallower depths to which each particular tissue can ascend and still remain within its empirically determined limit to avoid the bends.

Upon completion of the first six weeks of the course, students had developed an operational dive computer capable of computing no-decompression time limits for safe diving as well as staged decompression for profiles that exceeded these safe time limits at depth. This complex algorithm utilized numerical methods for root finding and numerical integration to track the nitrogen tissue pressure in a parallel arrangement of 17 tissue compartments while computational techniques for numerical differentiation and curve fitting were used to determine the tissue properties. The algorithm has also been tested for a broad range of diving scenarios.

- *Design Assignment:* The next step in the course was to apply this computational algorithm to an integrated design project. In our case, we chose the development of a pre-planned log for a series of multiple dives. This is typically what you would ideally want to use a dive computer to do. However, since commercial dive computers operate passively, measuring the ambient pressure and determining the current tissue pressures on the fly, they are only used to determine if you are following an already previously logged profile. With the software readily in hand, students were given the task of simulating a series of five dives throughout a single day, a fairly common profile on boat dives and working dives, i.e. by Navy divers or oceanographic researchers. The diver was required to perform the following tasks:
    - Obtain soil samples at a depth of 130 feet.
    - Obtain samples from a shipwreck located between 60 and 70 feet. This task will take a minimum of 40 minutes.
    - Obtain photographs and/or video footage from above the wreck at depths between 20 and 40 feet.
    - The diver must make a standard safety decompression stop at 15 feet for

10 minutes

- The tasks must be completed within a twenty-four hour period is such a way that they could be repeated on successive days.

In addition to this task description, the students were provided with the following constraints:

- The tasks must be completed in five dives from the open water surface.
- The tasks must be completed with only a single mandatory decompression dive (other than the standard fifteen-foot stop).
- The diver has only five, standard SCUBA air tanks rated at 64.5 cubic feet of compressed air at 2500 psi internal pressure.
- The diver's air consumption rate is given by the expression

$$\frac{1}{2} p_{AMB} \; \frac{ft^3}{\min}$$

- The diver must return to the boat after each dive with at least 300 psi of air in their tank.

With these constraints the students were to plan the workday dive profile in some optimal fashion, including specification of the following design variables:

- The maximum depth time for each dive and corresponding descent and ascent rates (based on maximizing the total time at depth for the five dives together).
- The surface interval time between dives (based on allowing for only a single mandatory decompression dive).
- The specification of air tanks sufficient for the tasks.

The deliverables for the project included:

- An objective function used to define the optimal dive conditions.
- Specification of the final dive profile sufficient to log the dive *a priori.*
- A summary of the modeling and numerical analysis, including all assumptions.
- A consideration of the design iterations performed, including several near and around the optimal solution.
- All relevant plots, i.e. tissue pressure histories, depth profiles, air consumption, etc.
- One piece of additional research on either optimization or numerical

methods that was used in their design analysis.

III. Conclusions

This project-based approach to teaching application of numerical methods has proven a popular and successful development. Students enjoy the project-based learning style, while instructors enjoy the challenge of synthesizing a numerically-intensive design application in their area of expertise. Student feedback indicated the following additional conclusions:

- Students felt the design project fostered a better appreciation for combining the use of several numerical modeling techniques to satisfactorily solve a complex problem.
- Students tended to ask more direct questions and appreciate the utility of particular numerical methods when they experienced concrete situations in which the advantages (and disadvantages) of such methods were more evident.
- By working through the weekly assignments as submodules, and integrating these submodules to perform the final design optimization, students learned the value of breaking a computer problem into smaller subtasks.

This project-based approach to integrating numerical analysis into instruction of design engineering has proven to be popular with both students and faculty. Students enjoy knowing how their work will be applied to a real engineering problem, while faculty enjoy integrating the series of prescribed numerical methods into a complex design application in an area of interest to them.

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JOSEPH C. MUSTO

Joseph C. Musto is an Associate Professor and Mechanical Engineering Program Director at Milwaukee School of Engineering. He holds a B.S. degree from Clarkson University (Potsdam, NY), and an M.Eng. and Ph.D. from Rensselaer Polytechnic Institute (Troy, NY), all in Mechanical Engineering. His industrial experience includes engineering positions with Eastman Kodak Company (Rochester, NY) and Brady Corporation (Milwaukee, WI).

VINCENT C. PRANTIL
Dr. Prantil received his BS, MS, and PhD in Mechanical Engineering from Cornell University. His research interests lie in microstructural material modeling, finite element and numerical analysis. He was a senior staff member at Sandia National Laboratories in the Applied Mechanics and Materials Modeling departments for ten years. He joined the mechanical engineering faculty at MSOE in September 2000.